

Section 7.4 Consumer/Producer Surplus

Consumer surplus = money they were willing to spend but saved b/c able to get for cheaper.

Producer surplus = charge they were willing to sell item for, but they made more than bottom line.

Formula: Consumer Surplus

$$CS = \int_0^{\bar{q}} D(q) dq - \bar{p} \bar{q} = \int_0^{\bar{q}} (D(q) - \bar{p}) dq$$

\bar{p} , \bar{q} = selling price & selling quantity

$p = D(q)$ = Demand = q
* looking for $p = \dots q$

Formula: Producer Surplus

$$PS = \int_0^{\bar{q}} [\bar{p} - S(q)] dq$$

$S(q)$ = Supply

$p = S(q)$ = q
* looking for $p = \dots q$

Consumer Surplus

Ex: 1

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$$p = 15e^{-0.01q}$$

Find CS (consumer surplus) when you sell CD's for \$5.

Step 1

$$= D(q) \\ \bar{p} = \$5.00$$

Step 2

$$\text{so } \frac{5}{15} = \frac{15e^{-0.01\bar{q}}}{15}$$

Step 3

$$\frac{5}{15} = e^{-0.01\bar{q}} \quad ; \quad \ln \frac{1}{3} = \ln e^{-0.01\bar{q}}$$

Step 4

$$\frac{\ln(\frac{1}{3})}{-0.01} = \frac{-0.01\bar{q}}{-0.01} \quad ; \quad \frac{\ln(\frac{1}{3})}{-0.01} = \bar{q}$$

Step 5

$$109.86 = \bar{q}$$

$$\text{so } \bar{p} = \$5 \quad ; \quad \bar{q} = 109.86 \quad ; \quad D(q) = 15e^{-0.01q}$$

Step 6 Put #'s into formula

$$CS = \int_0^{\bar{q}} D(q) dq - \bar{p}\bar{q} = \int_0^{109.86} 15e^{-0.01q} dq - (\bar{p})(109.86)$$

$$= 15 \cdot \frac{1}{-0.01} e^{-0.01q} \Big|_0^{109.86}$$

$$- 549.306$$

$$= -1500e^{-0.01q} \Big|_0^{109.86}$$

$$- 549.306$$

$$= 999.99 - 549.306 = \$450.69 = CS$$

Producer Surplus

Ex: #2 pg 547

Need to have a \bar{p} ; \bar{q} ; $p = S(q) = \dots q$

Step 1 $\overset{q}{\cancel{20}} = 20\sqrt{p-4}$ ← Wrong form - need $(p = \dots q)$

b/c this is (q) she is willing to provide

* Solve for p to get $p = \dots q$ format.

$$\text{, so } \frac{q}{20} = \sqrt{p-4} \quad = \frac{q^2}{400} = p-4$$

$$\text{, so } \frac{q^2}{400} + 4 = p \quad * \text{ Proper format}$$

Step 2 We were given $\bar{p} = \$8$, now solve for \bar{q} by plugging in \bar{p} value to original equation.

$$\text{, so } \bar{q} = 20\sqrt{8-4} = 20\sqrt{4} = 40$$

Step 3 $\bar{p} = \$8$; $\bar{q} = 40$; $S(q) = \frac{q^2}{400} + 4$

Plug into formula: $PS = \int_0^{\bar{q}} (\bar{p} - S(q)) dq$

$$= \int_0^{40} 8 - \left(\frac{q^2}{400} + 4\right) dq = \int_0^{40} \left(8 - \frac{q^2}{400} + 4\right) dq$$

$$= \int_0^{40} \left(4 - \frac{q^2}{400}\right) dq \quad * \text{ Now integrate} *$$

Step 4 $\int_0^{40} \left(4 - \frac{q^2}{400}\right) dq = 4q - \frac{1}{400} \frac{q^3}{3} \Big|_0^{40}$

Step 5 Plug into calculator to solve
2nd Trace $y = 4 - \left(\frac{q^2}{400}\right)$

2nd Trace # 7

Lower limit = 0 ; Upper limit = 40

Enter = $\boxed{\$106.67}$ Producer Surplus

Ex: #3 Equilibrium
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Step 1 Find equilibrium number

Supply = demand

$$S(q) = D(q)$$

$$p = \frac{q^2}{400} + 4 \quad q = \sqrt{200(16-p)}$$

↑

was previously
 $q = 20\sqrt{p-4}$

, so easier to use
this

$$\text{Step 2} \quad 20\sqrt{p-4} = \sqrt{200(16-p)}$$

square both sides to get rid of radicals, so

$$400(p-4) = 200(16-p)$$

$$400p - 1600 = 3200 - 200p$$

Step 3 Clean up algebra + get (p) by itself

$$\frac{600p}{600} = \frac{4800}{600}$$

$p = \$8$ charge to meet equilibrium

Step 4 Find (q) by plugging $p = \$8$ into one of the equations

$$\text{, so } 20\sqrt{8-4} = 20\sqrt{4} = 40$$

Step 5 So you need to sell 40 shirts @ \$8

$$\text{Step 6 CS} \Rightarrow \begin{aligned} \bar{p} &= \$8 \\ \bar{q} &= \$40 \\ \text{Req} &= \dots q = \end{aligned}$$

, so now take $q = \sqrt{200(16-p)}$ * solve for p

$$q^2 = 200(16-p) \quad \text{; } \frac{q^2}{200} = 16-p$$

$$\frac{q^2}{200} = 16-p \quad \text{; } \frac{q^2}{200} - 16 = -p \quad \text{;}$$

$$\left| 16 - \frac{q^2}{200} = p \right|$$

Step 7 now we have $\bar{p} = \$8$
 $\bar{q} = 40$

$$D(q) = 16 - \frac{q^2}{200}$$

Step 8 Plug into formula:

$$CS = \int_{\emptyset}^{\bar{q}} D(q) dq - \bar{p} \bar{q}$$

$$= \int_{\emptyset}^{40} 16 - \frac{q^2}{200} dq - (8)(40)$$

Step 9 Plug into calculator

$y = \dots$; 2nd Calc ; #7 ; Lower limit = \emptyset

Upper limit = 40 ; Enter = 533.33

$$= 533.33 - (8 * 40) = 533.33 - 320 = \text{\$}213.33$$

= CS

= 106.67 Producer Surplus

150 $213.33 + 106.67 = \text{\$}320.00$

From Ex #3 From Ex #2

~~106.67~~ $\text{\$}320.00$
Total social gain

Continuous Income Streams